Question 1 - Mathematical Induction

Use the principle of Mathematical Induction to show that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$.

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Question 2 - Mathematical Induction

Use Mathematical Induction to prove that $3^{2n} - 1$ is divisible by 8 for all natural numbers n.

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Question 3 -

Complex Numbers:

Let a be the last digit of your student number. For the next three parts of this question, you can use 1 decimal precision whenever necessary.

Write the complex numbers $z_1 = 1 + (a + 1)\,\mathrm{i}$ and $z_2 = -\frac{1}{2} + (a + 2)\,\mathrm{i}$ in polar form, and locate them in the complex (Argand) plane. In your diagram indicate the real axis, the imaginary axis, the modulus and arguments of both complex numbers

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Question 4 -

Complex Numbers:

Let a be the last digit of your student number. For the next three parts of this question, you can use 1 decimal precision whenever necessary.

Use the previously obtained z_1 and z_2 to compute $z_1^2 \cdot z_2^3$.

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Question 5 -

Complex Numbers:

Let a be the last digit of your student number. For the next three parts of this question, you can use 1 decimal precision whenever necessary.

Use the previously obtained z_1 and z_2 to compute $\frac{z_1^3}{z_2^4}$.

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Question 6 - Complex Numbers

Sketch in the complex plane all solutions of the equation $|z+2\mathrm{i}| = |z-3\mathrm{i}|$.

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Question 7 - Limits

Prove using the ε-δ definition of limit that $\lim_{x\to-1}(3x+1) = -2$.

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Question 8 - Continuity

Suppose that the function $f(x)$ is continuous on the closed interval $[1, 5]$ and that the only solutions of $f(x) = 2$ are: $x = 1$ and $x = 4$. If $f(2) = 4$, explain why $f(3) > 2$.

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Question 9 - Differentiation rules

Let n be all the digits of your student number. Find the n-th derivative of $f(x) = x \cdot e^{-x}$.

Hint: prove first that $f^{(k)}(x) = (-1)^k \cdot (x - k) \cdot e^{-x}$ for any natural number $k \geq 1$.

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